Stress Distribution and Possible Stress Induced Failure Around Roadway Tunnel inside Madhyapara Hardrock Mine, Bangladesh

Islam, M.S1, Chowdhury, Q2, Saumitra Narayan Deb3, Mashkovtsev I.L.4

Abstract
In the present research the detail stress analysis has been carried out using Kirsch’s equation inscribing circular roof of the tunnel. A computer program using FORTRAN has been developed in order to calculate the stresses in and around the tunnel opening. It is observed from the analyses that the maximum principal stresses are found at the corner of the arc of the opening. At the corner compressive stress is active whereas tensile stress is prevails on the top of the roof. We compared the different stresses whose are active surrounding the Roadway tunnel with strength of the rock and try to predict the possible failure. Strength of rock of different categories is determined from laboratory tests and found that the rock strength are 105 Mpa, 70 Mpa and 40 Mpa of rock categories I, II & III respectively. It is mentioned that rock are categorized on the basis of joint spacing. According to the stress distribution, the stress-induced failure could be occurred around the corner of the arc at initial period of tunnel construction. The line of failure was determined using uniaxial compressive strength of rock and the maximum and minimum principal stresses. It is found that notches and failed region are larger in rock category-III.

Key Words
Stress, Uniaxial compressive strength, Notch, Achaean basement complex, Madhyapara, Bangladesh

1. Lecturer, Department of Petroleum & Georesources Engineering, Shahjalal University of Science & Technology, Sylhet-3114, Bangladesh
2. Associate professor, Department of Geology & Mining, Rajshahi University, Rajshahi-6205, Bangladesh
3. Associate professor, Department of Mining & Petroleum Engineering, Peoples’ Friendship University of Russia, Moscow, Russia, sdeb@mail.ru
4. Professor, Department of Mining & Petroleum Engineering, Peoples’ Friendship University of Russia, Moscow, Russia

Introduction
The Madhyapara Hard rock Mine (MHM) is the first experience of hard rock mining and the second major mining project in the country. The Madhyapara Hard rock Mine situated in Parbatipur Upazila of Dinajpur districts in the northern part of Bangladesh. Stress is induced around tunnel due to mainly overburden pressure and hydrostatic pressure from different sides. When the stress larger than the strength of rock mass then
The different types of stresses like (maximum principal stress ($\sigma_1$), minimum principal stresses ($\sigma_3$) radial stress ($\sigma_r$), tangential stress ($\sigma_t$), and shear stress ($\lambda_{rt}$)) of different location along Roadway Tunnel on Madhyapara Hard rock Mine were calculated in the present research. The South-South Cooperation Corporation (NAMNAM) is the Mine construction and development company of North Korea working in MHM has classified total rock mass in mine into three categories (Category-I for spacing >2m, Category-II for spacing between 0.5 to 2.0m and Category-III for spacing <0.5m). Rock samples of different categories with different locations were collected and finally had been tested its strength like Uniaxial compressive strength. The stresses around tunnel was calculated by use of Kirch (1898) with FORTRAN programming and Surfer Programming. Stress induced failure was assessed by Hoek and Brown (1980) equation of failure. Finally notches and failed zones had been drowning. It is found that notches and failed zones around tunnel in rock category-III are remarkable. Considering the tectonic setting and the possibility of stress-induced failure, support in rock category-III is strongly suggested.

Regional and Geologic Setting

The Madhyapara Hard rock Mine (MHM) is the first experience of hard rock mining and the second major mining project in Bangladesh. The Madhyapara Hard rock Mine (MHM) situated in Parbatipur Upazilla of Dinajpur districts in the northern part of Bangladesh and lies between the latitudes $25°32'44''$N and $25°37'34''$N and the longitudes $88°59'09''$E and $89°06'48''$E (figure 1).
Madhyapara Hard rock Mine Project area is located in Rangpur Saddle. Rangpur Saddle is a possible connection between Indian Platform and Shillong massif and bounded by N-S trending faults (Figure 2). Based on geophysical prospecting, in 1974-76, the Geological Survey of Bangladesh (GSB) located hard rock Archean Basement Complex comprises of Quartz-diorite, granodiorite, gneiss, amphibolites etc. The depth of basement complex at Madhyapara is about 150m.
Interpretation of regional Bouger anomaly and magnetic anomaly maps gave a new idea of subsurface geological condition of the area. Interpretation of gravity data revealed that the structural set up of the basement rock is mainly fault controlled producing horst and graven. Graven and basin like structure is mainly reflected by gravity lows where gravity high in the Bouger anomaly maps generally due to the upliftment of the basement (figure 3). Large negative anomalies in NW part of the Bangladesh are probably related to the Basement features (Khan and Azad, 1962). Second derivative map of gravity supports the presence of its faults having varying throws and orientation in figure 4. Fault F2-F2 and F1-F1 are the most significant and in the sense that they controlled the southern and eastern boundary of MHM are respectively, where as F7-F7, F10-F10, F11, F11and F3-F3 have given rise to the graven type of structure in western side of the study area near Phulbari area.

![Figure 2: Tectonic map of Bangladesh and its adjoining area. (D.K. Guha, 1978)](image)

![Figure 3: Depth Contour map of Bedrock interface (NAMNAM, 2001)](image)
Review of Literature and Collection of data

The shape of excavation controls the stress distribution around an excavation in an elastic rock mass. Opening with corners or small radius of curvature show high concentration of compressive stress. Hence, there is a tendency to increase the radius of curvature in the design of underground openings, to avoid overstressing of the rock mass. In civil engineering practices tunnels are generally circular or horseshoe shaped. In mining practice, development of tunnel usually rectangular shapes with slightly arched roof which reduce the stress concentration. (Martin, et.al., 2000).

![Figure 4: Gravity based structural interpretation of Madhyapara and adjoining area (Modified after Rahman, 1984)](image)

The strength of a rock mass is often estimated by back calculation, where examples of failure have been carefully documented (Sakurai, 1993). In brittle rock masses failure around the tunnels occurs in the form of spalling or fracturing. Ortlep et al., (1972) compiled experience from square 3 to 4m tunnels in brittle rocks in South African gold mines which was suggested that the stability of these tunnels could be assessed using the
ratio of the far-field maximum stress ($\sigma_1$) to the uniaxial compressive strength ($\sigma_c$). A characteristic of the stress-induced failure in brittle rock is the notched-shaped failure, which is associated with slabling and spalling. These slab ranges from a few millimeters to ten of centimeter in thickness and with large openings can several square meters in surface area (Ortlep, 1997; Martin et al. 1997).

According to Martin et al. (1997), the slab formation is associated with the advancing tunnel face, and that once plane-strain conditions are reached the new notch-tunnel shaped is essentially stable. The mentioned knowledge about failure were used in the present analysis. Geological, hydrogeological and engineering data were collected from the boreholes and underground Roadway levels (production level, sub-level and ventilation) from different agencies and departments like GSB, NAMNAM etc. The different previous research work and different technical papers and analyses records were consulted for the present research work.

**Laboratory Works and Field Investigation**

Geo-engineering data like fault, fracture, fissure etc. and samples from different locations in Roadway tunnel level of Madhyapara Hardrock Mining Project (including core sample) are collected and mechanical properties of rock materials (like uniaxial compressive strength) are tested in the Road and Highway laboratory of Rajshahi University of Engineering and Technology. Cutting and polishing of samples are done in the lab of the department of Geology and Mining, University of Rajshahi. Several investigations were carried out in and around the mine area.

**Sequence of Study**

1. Collection of rock samples from different location along the Roadway level of Madhyapara Mine.
2. Two different types (block and cylindrical shapes) of sample were prepared for testing of Uniaxial Compressive strength.
3. Stress analysis: It is assumed that inscribing roof of tunnel is circular or arc of a circle. By using FORTRAN Programming, radial stress ($\sigma_r$), tangential stress ($\sigma_t$), shear stress ($\lambda_{rt}$), maximum principal stress ($\sigma_1$), and minimum principal stresses ($\sigma_3$) were calculated stresses around the periphery of the Roadway tunnel. For showing the stress distribution around the tunnel the Kirch (1998) equations are used.
4. $\sigma_1$ and $\sigma_3$ values had been drown by using the Surfer programming.
5. Possible Failure boundary demarcation: Failure boundary is marked by using Hoek and Brown’s (1980) equation.
Stress Analysis

The stresses, which exist around the opening in undisturbed rock mass in underground, are related to the weight of overlying strata and geological condition of the rock mass. This stress field is disturbed by creation of underground excavation of tunnel and others. So stress analysis is performed to know the stress distribution around the opening. In some cases, the distribution of stresses is high enough to exceed the strength of rock. In these cases, failure of the rock adjacent to the boundary can lead to instability, which may take the form of gradual closure of excavation, roof fall and slabling of sidewall.

In underground opening or tunnel, the rock mass around its periphery is under pressure or stress. There are two stress namely vertical stress and lateral stress. The vertical stress is the overburden load and the lateral stress is mainly due to hydrostatic pressure. The vertical stress is equal to the product of overburden thickness and unit weight of rock mass (20-30KN/m3). The stress distributions that can occur around the Roadway tunnel in Madhyapara Hardrock Mine (MHM) and in order to understand the mechanics of stress-induced instability and possible failure zone around the tunnel are studied in this paper.

Application of Kirch equation

The stresses around the opening are depends the depth and the mechanical properties of elastic materials. This is solved by the theory of elasticity. This requires that a set of equilibrium and displacement equations be solved for given boundary conditions and constitutive equations. The process involved in obtaining the solutions can become quite complex and tedious. One of the earliest solutions for the two-dimensional distribution of stresses around the opening in an elastic body was published in 1898 by Kirsch for the simplest cross-sectional shape, the circular hole. A full illustration of the Kirsch equations are given in figure 5. The final equations using a system of polar co-ordinate in which the stresses are defined in the terms other traction acting on the face of the element located by the radius r and polar angle $\theta$. 

Vertical stress

![Diagram of vertical stress](image-url)
Horizontal stress

**Figure 5**: Stress distribution around the circular tunnel (Hoek and Brown, 1980)

The vertical stress and horizontal stress are calculated by the following formulas:

Vertical stress \( (P) = \text{Depth of tunnel} \times \text{Rock unit weight} \) \( - \ ...) \quad (1) \)

Horizontal stress \( (P_1) = K \times P \) \( - \ ...) \quad (2) \)

Here \( k \) is the far field stress (the ratio of horizontal and vertical stress).

Tangential stress \( (\sigma_\theta) \), radial stress \( (\sigma_r) \) shearing stress \( (\lambda_{r\theta}) \) and are calculated by the following Kirsch equations with inscribing circular roof:

\[
\sigma_\theta = 0.5((1+K)(1-a^2/r^2)+(1-K)(1-4a^2/r^2+3a^4/r^4 \cos 2\theta)) \quad - \quad (3)
\]

\[
\sigma_r = 0.5((1+K)(1+a^2/r^2)-(1-K)(1+3a^4/r^4 \cos 2\theta)) \quad - \quad (4)
\]

\[
\tau_{\theta r} = 0.5(-(1-K)(1+2a^2/r^2-3a^4/r^4) \sin 2\theta) \quad - \quad (5)
\]

Where, \( a \) is the radius of the opening and \( r \) is the radial distance of any point away from the center of opening.

Principal stresses in plane of paper at point \((r,\theta)\) are calculated by using the above equations 3, 4 and 5:

\[
\sigma_3 = 0.5(\sigma_r + \sigma_\theta) - (0.5(\sigma_r - \sigma_\theta)^2 + \tau_{\theta r}^2)^2 \quad - \quad - \quad - \quad - \quad (6)
\]

\[
\sigma_1 = 0.5(\sigma_r + \sigma_\theta) + (0.5(\sigma_r - \sigma_\theta)^2 + \tau_{\theta r}^2)^2 \quad - \quad - \quad - \quad - \quad (7)
\]

Where, \( \sigma_1 \) is the maximum principal stress at failure, \( \sigma_3 \) is the minimum principal stress at applied field.
a is the radius of tunnel, r is the radial distance from the center of the tunnel and k is far field stress ratio.

Stresses \( (\sigma_\theta, \sigma_r, \lambda_r\sigma_r) \) on the boundary and outside of the opening are calculated by changing values of \( r \) and \( \theta \). The manual process of calculation is very much tedious. So, a FOTRAN program were built to calculate.

**Stress distribution in Madhyapara**

Calculation of stress distribution is an important in Madhyapara Hard rock Mine area. The cross-section in the study area is arched shape. The stress distributions are influenced due to arched effect of roof of the tunnel. Though we calculate stresses for different values of \( k \), like 0.0, 0.4, 1.0, 1.4, and 1.8 (Table 2) but finally \( k = 0.5 \) is used (figure 6). Hoek and Brown are use this value for the most cases of stress analysis especially for medium strength rock at shallow depth (<500m). If the radius of opening is \( r \), then the zone of influence will be equal to 3\( r \). Considering this, stresses at different location on the periphery and other points outside of the opening are calculated. After getting the values of stresses \( (\sigma_1, \sigma_3) \) at different location contours are drawn by a contour drawing program Surfer.

**Result and Discussion**

It is revealed from the different figures of 6 that the maximum principal stresses more higher at corner of the opening than other points. The reverse condition is prevailed for minimum principal stresses for different categories of rock.
Stress in rock category-I
It is found from figure 6 (a) that compressive force is active at the corner while tension force is acting at the top of the roof of the opening. The values of principal stresses ($\sigma_1$ and $\sigma_3$) are ranges from $4.5$-$10$ M Pa and $0.5$-$4.5$ M Pa respectively. From the above discussion it is clear that little stress-induced failure could occur at roof and at corner of the opening.

Stress in rock category-II
It is found from figure 6(b) that compressive force and tension force are active in same direction as in rock category-I. The values of principal stresses ($\sigma_1$ and $\sigma_3$) are ranges from $4.5$-$10$ M Pa and $0.5$-$4.5$ M Pa respectively. From the given figure 6 (b) it is said that little stress-induced failure could occur at roof and at corner of the opening.

Stress in rock category-III
Found that compressive force and tension force are active same direction as in rock category-I. The values of principal stresses ($\sigma_1$ and $\sigma_3$) are ranges from $5.0$-$12$ M Pa and $0.0$-$4.5$ M Pa respectively. From the given figure 6(c) it is said that large stress-induced failure could occur at roof and at corner of the opening.

Figure 6: Stress distribution around the tunnel for different rock categories (a), (b) and (c)
Possible Stress induced failure in Madhyapara Hard rock Mine

The critical stress-intensity factor represents the ability of the material to withstand a given stress-field intensity at the tip of crack and resist tensile crack extension. Brittle fracture is a sudden catastrophic failure, which occurs suddenly without prior plastic deformation and can occur nominal stress levels below yield. Brittle fracture becomes more predominant as member thickness, constraint and loading rates increase and as temperature decreases. There are many research carried out by many workers in this field but nobody aware of failure criterion which meet all required failure conditions. There are many available failure theories offer an excellent explanation for the aspects of rock behavior but failed to explain others or cannot be extended beyond a limited range of stress conditions. Consequently, faced with the task of providing a failure criterion which will be of practical value of underground excavation designers, Hoek and Brown (1980) seek to provide a new criteria which would meet at least most of the required failure condition. They have drawn on their experience in both theoretical and experimental aspect of rock behavior to develop by a process of trial and error, the following empirical relationship between the principal stresses associated with the failure of the rock.

\[ \sigma_1 = \sigma_3 + \sqrt{m \sigma_3 \sigma_c + s \sigma^2} \]  

Where \( \sigma_1 \) and \( \sigma_2 \) are maximum and minimum principal stresses, and \( \sigma_c \) is the uniaxial compressive strength of rock (laboratory result), and \( m \) and \( s \) are Hoek and Brown’s parameter. The following values of \( m \) and \( s \) are used on the basis of rock categories.

From the stress distribution in previous discussion is found that there would be failed in some places around the roof of the tunnel. The zone of failure are calculated by the Hoek and Brown’s (1980) Failure criteria equation (8)- Maximum stress \( \sigma_c \) at point in which the tunnel could fail, are compared with laboratory test result. The approximate equal values of maximum stress \( \sigma_c \) to laboratory result are defining the boundary of failed zone.

Therefore

\[ \sigma_c = (m \sigma_3 \pm \sqrt{m^2 \sigma_3^2 + 4sx(s_3 - \sigma_3)^2}) / 2s \]  

- - - - (9)

**Determination of Notch**

Notches are determined by using the stress distribution around the opening. From the stress distribution and comparison of \( \sigma_1 \) and \( \sigma_3 \) to \( \sigma_c \) Notches have been drown. It is found that Notches are mainly developed at the top of the opening. Maximum boundary limit or top of the Notch is at a point from where stress-induced failure could occur. Notches are illustrated in different figures 7.

Determination of failed zone:

Stress-induced failed zones are determined by the aforesaid failure equation (9). To comparison of \( \sigma_1 \) and \( \sigma_3 \) to \( \sigma_c \) the possible boundaries of failed zones are marked. Failed zones are illustrated in different figures 7.

**Discussion and Conclusion**
It is found from the figure 7(a) that the possible shape of notch in rock category-I is small compared to its radius. Notch and possible failed region due to overstressing condition in rock category-II is remarkable which is illustrated in figure 7(b). The notches formed in rock category-III are very much remarkable. Notches are formed on the top of the tunnel. Failed region in overstressing condition is very large compared to of others rock categories.

**Rock category-I**

![Diagram](image1)

**Rock category-II**

In the present research the detail stress analysis has been carried out using Kirsch’s equation inscribing circular roof of tunnel. A computer program using FORTRAN has been developed in order to calculate the stresses in and around the tunnel opening.

Stresses on roof of tunnels like radial stress ($\sigma_r$), tangential stress ($\sigma_t$), shear stress ($\lambda_{\theta\theta}$), maximum principal stress ($\sigma_1$) and minimum principal stress ($\sigma_3$) have been calculated by Kirsch equation. Stress values are obtained for different k (far field stress ratio) values but finally, the value of k is considered 0.5. Hoek and Brown were used this value for shallow depth (<500m) rock and medium strength rock. Failure criteria is valid only for k=0.5. It is observed from the research that stress distribution along the Roadway tunnel is important. Tunnel could fail for different causes. The prime cause is stress-
induced failure. Stress distribution is depended on shape, size and curvature of the roof etc. By using Kirsch’s equation stress on roof of tunnel is calculated and found that the maximum principal stress ($\sigma_1$) is higher at the corner of arched or contact of sidewall and roof of the tunnel.

It is showed that little or no stress-induced failure could occur in category-I and category-II. It is also showed that in category-III, stress-induced failure region is large. Studies on installment of supports in category-III to protect stress-induced failure are strongly suggested.

References


### Appendices

#### Tables

**Table 1: Used values of m and s**

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<th>Rock category</th>
<th>Value of m</th>
<th>Value of s</th>
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<td>Category-III</td>
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*(Ref. Hoek and Brown, 1972, *Underground Excavation in Rocks*, p 205)*

**Table 2: Data of $\sigma_1$ and $\sigma_3$ to determine $\sigma_c$, for the determination of failed areas in different rock categories**

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102
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