Relationship Marketing replicates Small-World Experiment

Sarat C. Das
KiLax Ltd, London, UK

Keywords
Small-world experiment, Six Degrees of Separation, common pathways, metabolic network, average path-length, clustering co-efficient

Abstract
Relationship Marketing, which is at the heart of marketing management, inherits a small-world-type network characterized by short path-lengths. At a strategic level the discipline can gain from modelling on Stanley Milgram’s (1967) small-world experiment on average path length for social networks.

The relationship marketing can consciously map the ‘average number of steps’ along the shortest paths for all possible pairs of social network nodes to configure a model that can measure efficiency of achieving a certain response from the task of information-sharing emphasising on customer retention and satisfaction and even a transaction value. These efforts are expected to design innovative collaborative and social communication channels to push the agenda of relationship marketing by developing and implementing appropriate marketing plan to stimulate referrals (referral marketing). It also can create certain demographic and customer service data pervading all the spaces of referral marketing covering internal markets, influence markets, and customer markets and supplier markets. Thus, the sub-markets, for example government regulators and consumer associations falling under influence markets, can be better targeted with a laser-precision.

The research explores the complexities in designing such a model based on Milgram’s (1967) small-world experiment combined with other inspired models such as Metcalfe’s law, Scott Feld’s friendship paradox, Kevin Bacon’s six degrees of separation or a ‘material-semiotic’ method developed by Bruno and Callon which come to be known as ‘actor–network theory’, or even the Erdos number which measures the collaborative distance between individuals.

1. Introduction

Small world draws an interesting parallel with the social structure of relationship marketing. The commodity exchange, which is at the core of relationship marketing, can trace its origin to gift exchange in ancient world; it is an emerging social anthropological perspective that interprets the relationship marketing theories and practices of today. In essence the gift-exchange may not be similar to barter or exchange appropriated to a monetary value but it operates within a remit of a social network identical to the latter. The early research into the gift economies was pioneered by Bronislaw Malinowski’s description of the Kula ring1 in the Trobriand Islands during First World War; kula exchange or kula ring is a ceremonial exchange system conducted in the Milne Bay Province of Papua New Guinea near Australia continent. The essence of gift economy is fostering relationship in a small community, or so called the small world. As the society expanded the contours of the small world were redefined.

Dunbar (1993: 681) tries to determine the group size in some of the contemporary small-worlds, and he finds the group size covaries with relative neocortical volume in nonhuman primates. He develops a regression model to predict a group size of the current small-world to that of hunter-gatherer and traditional horticulturalist societies. Social grooming time is linearly related to group size as social bonding is inversely proportional to efficient method for time-
sharing, not ignoring the role of language in the same: 'maintaining stability of human-sized groups by grooming alone would make intolerable time demands'.

The group size relates of Neolithic society relates to the circle of acquaintances of the contemporary society. Dunbar's claim of the upper limit of a group size's dependency on the neocortex's information-processing capacity (cognitive constraint) to maintain cohesion and integrity of groups may seek a new interpretation in the contemporary digital society. The average path length in a social relationship is no more modelled on physical medium because of other co-existing mediums such as Internet or telephone. The latter mediums have created their own measurement of distance based on both randomisation and ordering. It would be a fallacy to say the chance meeting and fostering relationship over Internet is always at random and can never be predicted. In a physical world the access to internet technologies and clustering of these access points seem to have created a very determined system. Even it may look there is a random dispersement of surfers in the Internet world these netizens create their circle of acquaintances or so called networks, and further these networks spread with their interconnections through group interbreeding. The foundation of these relationships is identical to small-world network.

The understanding of an existing network in a small world recalls Stanley Milgram’s iv anecdote scripted in 1967:

Fred Jones of Peoria, sitting in a sidewalk cafe in Tunis, and needing a light for his cigarette, asks the man at the next table for a match. They fall into conversation; the stranger is an Englishman who, it turns out, spent several months in Detroit studying the operation of an interchangeable-bottlecap-factory.

"I know it's a foolish question," says Jones, "but did you ever by any chance run into a fellow named Ben Arkadian? He's an old friend of mine, manages a chain of supermarkets in Detroit..."

"Arkadian, Arkadian," the Englishman mutters. "Why, upon my soul, I believe I do! Small chap, very energetic, raised merry hell with the factory over a shipment of defective bottlecaps"

"No kidding!" Jones exclaims in amazement.

"Good lord, it's a small world, isn't it?" (Milgram:61)

The relationship marketing today replete with many such anecdotes. Chance meetings or chance encounters can merely be said to be random variables emanating from a random process; these social relationships represent the evolution of some random value, or system, over time studied under stochastic process (Ross, 1996). iv The stochastic process is putatively known as the probabilistic counterpart to a deterministic process (deterministic system). Instead of describing a process, a relationship in marketing, which can only evolve in one way, in a stochastic or random process there is some indeterminacy: even if the initial condition is known (the customer steadfastly spends a certain sum or buys certain products or prefers physical shopping than ordering over telephone or Internet), there are several directions in which the process may evolve (such as changing buyers preferences for products or mode of purchase or change of circumstances).

2. Current Relevance

At the core of any society is the bonding and time spent on nurturing the relationship. The rise of consumerism following the World War II created its own unique societies, multi-layered,
interbreeding social groups which continuously evolve by themselves. Yet as always, the world remained as a cluster of small-worlds due to neocortical constraint.

Relationship marketing in this context is a social phenomenon but simultaneously a mathematical expansion. There is a continuous effort to connect the remote – expansion from the immediate priority customers to future customers. How improbable some individuals may appear to be a part of an organization’s market territory at present but they cannot be ignored to be included among the future possibilities. Imagine a bus load of overseas tourists passing through the habitats of Jarawa tribes of India’s Andaman Islands creating trading opportunities, not ignoring India’s Supreme Court’s ruling restricting human-contact. In another instance, a field anthropologist may occur to an idea to trade his or her watch with an aborigine’s bow and arrow.

Pool and Kochen (1978) vi purported a model of two individuals at random connected with ‘k’ intermediaries assuming a synthetic, homogenous structure. Subsequently Rapaport and Horvath (1960) vii put forward an empirical study on school friendships claiming asymmetric nets and the Universe is small. These small-world experiments post War paved the way for a more robust Milgram’s empirical validation of the concept known as small-world experiment. At the heart of this experiment is to establish a probability function that two randomly chosen individuals would know each other. Both can be connected through an average path length which can be mapped on a social network.

The experiment set Milgram, which perhaps not inconceivable to an innovative form of a mail order advertising conditioned by relationship marketing, to send out a chain-letter, a folder shape document he called 'letters of solicitation', through which individuals were randomly chosen across geographies wide apart from each other. The folder contained 'tracer' cards allowing an individual to return to Milgram every time the person advanced the packet to whom he or she knew on a first-name basis. The experiment found the distance between the 'starter' to 'target' individual is a limited number of 'immediate acquaintances' (the chains varied from two to 10 intermediate acquaintances) with the median at five. viii

Milgram's empirical studies found that any individual thus connected to the other with an average path length of six jumps. Many precursors of Milgram popularised this phenomenon – playwright John Guare named a play 'Six Degrees of Separation', Kevin Bacon's parlour game based on his own assertion that he had collaborated with all in Hollywood directly or through someone. Statistiix (Dempster et al 1977) (Diebolt and Robert, 1994) believed that the interconnectedness of individuals can be mapped on the path of a social network and further invested in social capital. Statist designed many Eastern European cities, set his rules of thumb on this belief. It appears the human beings are always predisposed to the notion of a 'small world'. Milgram postulates that any two individuals are connected through a series of links which he terms as 'immediate acquaintances'. His experiment unravels a certain mathematical structure underpinning a societal formation.

Milgram purports three philosophical views on the small world problem:

1. any two individuals in the world, no matter how remote from each other, can be linked in terms of intermediate acquaintances, and such intermediate links is relatively small.
2. There is an unbridgeable gaps between various groups, terminating the possibility of two people in the world ever be linked up, because they have circles of acquaintances who do not necessarily intersect.
3. Combines the two above views: We live in a world with deep social cleavages, consistent with all the empirical research to follow on the "small world problem."
Milgram’s conceptions of these interconnectedness and relationship-driven world is quite pictorial. See below:

**Fig 1 (Milgram, May 1967)**

Random dispersion of people in the small world.

**Fig 2. (Milgram, May 1967)**

Each person’s first-hand acquaintances are shown. A through N.

**Fig 3 (Milgram, May 1967)**

The network spreads, with complicated inter-connections.

**Fig 4. (Milgram, May 1967)**
Milgram reflects on following posers

- One way of restating the small-world problem: Given any two of these points chosen at random from this universe ... through how many intermediate points would we pass before the chosen points could be connected by the shortest possible path?
- The odds drop sharply on chances of knowing mutual acquaintance compared to ‘first-name’ or first-hand acquaintance. Pool-Kochen corroborates the claim stating a 50-50 chance that any two individuals can be linked up with two intermediate acquaintances.
- Many from the circle of acquaintances feedback into the same circle of acquaintances. Thus, the contours of social structure not built on random connections among persons but tend to be fragmented into social classes and cliques. This aspect of fragmentation can be particularly contextualised in relationship marketing. After all the practice of relationship marketing has been facilitated by several generations of customer relationship management, the same way fragmentation engenders social classes and cliques, each of this social group has its own litany of preferences, activities, tastes, likes, dislikes, and complaints.

3. Common Pathways

The creation of social classes and cliques give birth to common pathways. Each of these pathways is equally inclined to feed into an outside contact; also, there seem to be highly sought-after channels for the transmission of the chain. Second, there is differentiation among these commonly employed channels, so that some channels dispense the key points of transmission pertaining to residential contact while others have specialised contact possibilities in the occupational domain. For each possible realm of activity, in which the target individual is a part of, there is likely to emerge a sociometric star with specialized contact possibilities. (Milgram 1967)

These possibilities do bring about certain geographic and social movement, Milgram asserts and raises the following questions:

In what degree are the racial lines surmounted?
Can any sizeable fraction of the communications get through the racial barrier?
If the answer is yes, we then want to figure out the typical locus of transmission. (Milgram 1967)
4. Post-Milgram, Post-Beetle

Milgram’s conceptions of relationship pathways / interconnectedness become complex. For example, a knowledge society, a variant of post-industrial society, hereupon can be based on certain individuals’ creation of a social group in themselves by their certain privilege access to technology, e-governance, green economy, human capital development, social and humanistic computing (proliferation of Internet forums ranging from e-communities engendered by social network sites, collaborative groupware, web logs, etc.).

Watts and Strogatz (WS) \addtocounter{equation}{15} (Watts and Steven, 1998) conceived these avant-garde societies lying between extremes such as randomness and certain predictability:

*Networks of coupled dynamical systems have been used to model biological oscillators, Josephson junction arrays, excitable media, neural networks, spatial games, genetic control networks and many other self-organizing systems. Ordinarily, the connection topology is assumed to be either purely regular or random. But many biological, technological and social networks identify themselves somewhere between these two extremes. Here we explore simple models of networks that can be tuned through this middle ground: regular networks ‘rewired’ to introduce increasing amounts of disorder... these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. We call them ‘small-world’ networks, by analogy with the small-world phenomenon ... Models of dynamical systems with small-world coupling display enhanced signal-propagation speed, computational power, and synchronizability. In particular, infectious diseases spread more easily in small-world networks than in regular lattices.*

WS model\addtocounter{equation}{15} (Wu, 2007) substantiates Milgram’s model terming it as ‘small-world network’ containing regular and random networks. According to WS, in a regular network a point is directly linked to its four immediate neighbors while in a random network, each point has a connection to a distant point, can be quite far, reducing the number of links required to reach across the network. (Wu, 2007)

To interpolate between regular and random networks, we consider the following random rewiring procedure (Fig. 5). Starting from a ring lattice with \( n \) vertices and \( k \) edges per vertex, we rewire each edge at random with probability \( p \). This construction allows us to ‘tune’ the graph between regularity (\( p \approx 0 \)) and disorder (\( p \approx 1 \)), and thereby to probe the intermediate region \( 0 , p , 1 \), about which little is known.

![Fig 5: Source: (Watts, 1999) & (Wang and Chen, 2003)](image-url)
Because WS small-world model is that the connectivity distribution of a network peaks at an average value and decays exponentially. (Wang and Chen, 2003: 8) Such networks are called “exponential networks” or “homogeneous networks,” because each node has about the same number of link connections.

The mutations of the knowledge societies can be complex; which have engendered scale-free networks entailing connectivity distributions in a power-law form at a variance from an exponential network. These networks are inhomogeneous in nature: majority nodes have few link connections and yet a few nodes have too many connections. The discovery of scale-free networks was based on the observation that the degree distributions of many real networks have a power-law form, albeit power law distributions have been investigated for a long time in physics for many other systems and processes. (Wang and Chen, 2003)

The Barabási and Albert (BA model) suggest that two main ingredients of self-organization of a network in a scale-free structure are growth and preferential attachment. (Wang and Chen, 2003: 14) These point to the facts that most networks continuously grow by the addition of new nodes, and new nodes are preferentially attached to existing nodes with large numbers of connections (again, “rich get richer”).

The current focus of the networked relationships has been on the average path length, clustering coefficient, and degree distribution. It includes the WS construct of a network model with small average path length as a random graph and relatively large clustering coefficient as a regular lattice and also scale-free networks based on the observation that the degree distributions of many real networks have a power-law form. (Wang and Chen, 2003)

Fig 6 (Wang and Chen, 2003)
4.1 Average path length

A network topology which measures the average number of steps along the shortest paths for all possible pairs of network nodes and to be differentiated from the diameter of the network. Known as a measure of the efficiency of information on a network it is a mean distance between two nodes, averaged over all pairs of nodes. It was an interesting discovery that the average path length of most real complex networks is relatively small, even in those cases where these kinds of networks have many fewer edges than a typical globally coupled network with an equal number of nodes.\textsuperscript{xxiii} (Wang and Chen, 2003: 9)

4.2 Clustering Coefficient

Common circle of immediate acquaintances leads to clustering of the network -- the average fraction of pairs of neighbours of a node that are also neighbours of each other. The real-world networks particularly witnesses nodes that engender close-knit groups characterised by a relatively high density of ties; this probability tends to be more than the average probability of a tie randomly established between two nodes.\textsuperscript{xxiv} (Holland and Leinhardt, 1971)

4.3 Degree Distribution

The number of connections or edges the node has to other nodes defines the probability distribution of these degrees over the whole network. If a network is directed, meaning that edges point in one direction from one node to another node, then nodes have two different degrees, the in-degree, which is the number of incoming edges, and the out-degree, which is the number of outgoing edges. The degree $k$ of a node is usually defined to be the total number of its connections. Thus, the larger the degree, the “more important” the node is in a network. The average of $k_i$ overall $i$ is called the average degree of the network.\textsuperscript{xxv} (Wang and Chen, 2003: 10)

Table 1\textsuperscript{xxvi} (Wang and Chen, 2003: 10)

<table>
<thead>
<tr>
<th>Network</th>
<th>Size</th>
<th>Clustering coefficient</th>
<th>Average path length</th>
<th>Degree exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internet, domain level [13]</td>
<td>32711</td>
<td>0.24</td>
<td>3.56</td>
<td>2.1</td>
</tr>
<tr>
<td>Internet, router level [13]</td>
<td>228298</td>
<td>0.03</td>
<td>9.51</td>
<td>2.1</td>
</tr>
<tr>
<td>WWW [14]</td>
<td>15127</td>
<td>0.11</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>E-mail [15]</td>
<td>56969</td>
<td>0.03</td>
<td>4.95</td>
<td>1.81</td>
</tr>
<tr>
<td>Software [16]</td>
<td>1376</td>
<td>0.06</td>
<td>6.39</td>
<td>2.5</td>
</tr>
<tr>
<td>Electronic circuits [17]</td>
<td>329</td>
<td>0.34</td>
<td>3.17</td>
<td>2.5</td>
</tr>
<tr>
<td>Language [18]</td>
<td>460902</td>
<td>0.437</td>
<td>2.67</td>
<td>2.7</td>
</tr>
<tr>
<td>Movie actors [5, 7]</td>
<td>225226</td>
<td>0.79</td>
<td>3.65</td>
<td>2.3</td>
</tr>
<tr>
<td>Math. co-authorship [19]</td>
<td>70975</td>
<td>0.59</td>
<td>9.50</td>
<td>2.5</td>
</tr>
<tr>
<td>Food web [20, 21]</td>
<td>154</td>
<td>0.15</td>
<td>3.40</td>
<td>1.13</td>
</tr>
<tr>
<td>Metabolic system [22]</td>
<td>778</td>
<td>-</td>
<td>3.2</td>
<td></td>
</tr>
</tbody>
</table>
5. More Complex Networks, More Complex than Post modernism

Fig 7xxvii (Wang and Chen, 2003: 11)

5.1 Regular Coupled Networks

Intuitively, a globally coupled network has the smallest average path length and the largest clustering coefficient. Although the globally coupled network model captures the small-world and large-clustering properties of many real networks, its limitations can be pointed out at ease: a globally coupled network with $N$ nodes has $N (N −1)/2$ edges, while most large-scale real networks appear to be sparse, that is, most real networks are not fully connected and their number of edges is generally of order $N$ rather than $N^2$.

5.2 Random Graphs, Beyond Erdos and Rényi (ER)’s conception

A noteworthy discovery of this type was that key properties of random graphs can appear like a bolt out of the blue. Yet this logarithmic increase in average path length with the size of the network is a typical small-world effect.xxviii (Wang and Chen, 2003: 12). This means that a large-scale random network does not show clustering in general. In fact, for a large $N$, the ER
algorithm generates a homogeneous network, where the connectivity approximately follows a Poisson distribution.\textsuperscript{xxix} (Wang and Chen, 2003)

Fig 8\textsuperscript{xxx} (Wang and Chen, 2003)

5.3 Synchronization in Complex Dynamical Networks
Synchronization can be achieved in convoluted and constantly evolving networks such as witnessed in turing patterns, auto-waves, spiral waves, and spatiotemporal chaos. These networks are usually described by systems of coupled ordinary differential equations or maps, with completely regular topological structures such as chains, grids, lattices, and globally coupled graphs. Two typical settings are the discrete-time coupled map lattice (CML)\textsuperscript{xxxi} (Kaneko, 1992) and the continuous-time cellular neural (or more generally, nonlinear) networks (CNN)\textsuperscript{xxxii} (Chua, 1998).

5.4 Synchronization in Small-World Networks
It is possible to achieve synchronization in a large-size nearest-neighbour coupled network that can be greatly enhanced by just adding a tiny fraction of distant links, thereby making the network become a small-world model. This process reveals an advantage of small-world networks for attaining synchronization, if desired.

Fig 9,\textsuperscript{xxxiii} (Wang and Chen, 2003)
Kleinfeld points out the ‘small-world problem’ due to a mathematical demonstration of how random connectors in a network can create a small world.

Psychological research is required to examine the empirical realities and why people have strong emotional needs to believe we live in a small, small world. (Wang and Chen, 2003: 18)

6. Relationship Marketing Hotspots

Any form of relationship marketing will replicate a form of network model leading to a set of concerns of knowing how the network structure assists or constrains the network dynamical behaviours. For example, how do social networks mediate the word-of-mouth publicity, optimising a ‘marketing mix’ to the advantage of a product, how a task is allocated to a cross-functional team, or a re-engineering process is initiated affecting a particular design of workflow, etc. It is important to avoid the cascading failures propagating throughout a large grid of network.

For exponential networks, as it would matter in viral marketing, the epidemic threshold is a positive constant, for a large class of scale-free networks the critical spreading rate tends to zero. In other words, scale-free networks are prone to the spreading and the persistence of infections, viral messages would pervade regardless of the spreading rate of the epidemic agents. Yet the theory implies that computer viruses can spread far and wide on the Internet by infecting only a tiny fraction of the huge network. Fortunately, this is balanced by exponentially small prevalence and by the fact that it is true only for a range of very small spreading rates ($\lambda << 1$) that tend to zero. (Wang and Chen, 2003: 16)

The companies which are using complex networks may occur to situations of disappearance of the node implies the removal of all of its connections, disrupting some of the paths among the remaining nodes.

7. Conclusion

The relationship marketing poses huge challenges with regard to modelling, analysis, control, and synchronization of complex dynamical networks. The approaches to manage relationship networks require to balance between optimal growth and avoiding cascading network failures.

8. Research limitations and direction for further research

The small-world experiment conceived and the subsequent empirical studies were conducted in the pre-digital era. Since that time digital world has redefined geographical boundaries, created new experiences of consumerism and social cohesion and engendered new forms of human associations and sociability (Simmel's concept of indulging in the conversational and relational games that engender conviviality and shared experiment as opposed to be immersed in social structures and positionings related to hierarchies and disparity in social fields). (Jovchelovitch, 2011)

Earlier a small world experiment structure takes the shape of a cluster of approximately 200 million points (Travers and Milgram, 1969: 426) with a convoluted web of connections among them. The change of demographics combined with all the characteristics (impact-points of digital world) may have created more circle of acquaintances (acquaintance chains), thus different pathways, of various lengths, between any two individuals.
9. References


[18] Ibid.,


[20] Ibid.,
[25] Ibid.,
[28] Ibid.,
[29] Ibid.,
[33] Ibid.,
[34] Wang, Xiao Fan and Chen, Guanrong (First Quarter 2003) Complex Networks: Small-World, Scale-Free and Beyond, IEEE Circuits and Systems Magazine (Digital Vision) p 16