Optimal inventory policies for Weibull deterioration under trade credit in declining market

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Key-words
Weibull deterioration, trade credit, sensitivity analysis, inventory.

Abstract

The aim of this study is to develop mathematical model for Weibull deterioration of items in inventory in declining market when the supplier offers his retailers a credit period to settle the accounts against the dues. The computational steps are explored for a retailer to determine the optimal purchase units which minimizes total inventory cost per time unit. The numerical examples are given to demonstrate the retailer's optimal decision. A sensitivity analysis is carried out to study the variations in the optimal solution.

Introduction

Till early 80's, the inventory models were derived under the assumption that the retailer settles the account immediately on the receipt of the goods in inventory. Brigham (1995) gave term "net 30" which means a supplier offers 30-days time period to the retailer's to settle the account against the items procured. The supplier does not charge any interest for the dues if it is paid within the 30-days. However, if the payment is not settled within the 30-days, then interest is charged on the unsold stock in the retailer's inventory. The retailer can earn the interest on the revenue generated and delay the settlement of account till the last allowable date of permissible credit period by the supplier. Thus, by taking advantage of trade credit, the retailer can reduce his total cost, equivalently, trade credit is discounting. For the supplier, it may be default risk (Teng et al. (2005)).

The concept of trade credit inventory model was formulated by Goyal (1985). He discussed interest earned on the unit purchase price and concluded that the cycle time and order quantity increases marginally. Dave (1985) corrected the Goyal's model by assuming the fact that the selling price is higher than its purchase price. The interest earned by the retailer should be computed on the selling price. Shah (1993a) derived a mathematical model when units in inventory are subject to constant deterioration and tread credit is offered to the retailer by the supplier. Shah (1993b, 1993c) formulated the probabilistic inventory model under the assumption of permissible delay in payments. The order level probabilistic inventory model is derived for deteriorating items to study the effect of the permissible delay period in Shah (1993d). Hwang and Shinn (1997) developed the joint pricing and ordering policies for the retailer under the scenario of allowable trade credit. Liao et al. (2000) developed an inventory model when demand is stock-dependent. Most of the above stated articles and their references ignored the difference between unit sale price and unit purchase cost, concluding to the findings of Goyal (1985).

Jamal et al. (1997, 2000) and Sarker et al. (2000) took the difference between the unit sale price and unit purchase price to establish that the retailer should settle the account as sooner as the unit selling price increases relative to the unit purchase price. Teng (2002) provided an alternative conclusion that the saturated retailer should place order of smaller size to avail of the permissible delay more frequently. One can read articles by Abad and Jaggi (2003), Arcelus et al. (2001), Arcelus et al. (2003), Chung and Dye (2000), Chung and Dye (2001), Chung (1998), Chung (2000), Chung and Huang (2003), Chung and Liao (2004), Chung et al. (2005), Shah (2004), Shah (2006), Gor and Shah (2003), Gor and Shah (2005), Huang (2003), Ouang et al. (2005a, 2005b), Salameh et al. (2003), Shah et al.(2004), Shinn and Hwang (2003). The most of the above cited study is derived under the assumption of constant and known deterministic demand.

In this article, the demand for a product is assumed to be decreasing with time. The decrease in demand is observed for fashionable garments, seasonal products, air-tickets etc. Shortages are not allowed and replenishment rate is infinite. The units in inventory are subject to deterioration with time. It is assumed that deterioration follows two-parameters Weibull distribution. The retailer generates revenue on unit selling price which is necessarily higher than the unit purchase cost. The objective is to minimize the total cost per time unit of an inventory system. The model is supported by numerical examples. The sensitivity analysis is carried out to study the variations in the optimal solution.

Notations and Assumptions :

The proposed mathematical model is developed using the following notations and assumptions :

Notations

$R(t)$: = a(1-bt); the annual demand as a decreasing function of time where a > 0 is constant demand and b (0 < b < 1) denotes the rate of change of demand with respect to time.

C : the unit purchase cost.

P : the unit selling price with (P > C).

h : the inventory holding cost per unit per annum excluding interest charges.

A : the ordering cost per order.

M : the permissible credit period offered by the supplier to the retailer for settling the account against the dues.

I_c : the interest charged per monetary unit in stocks per annum by the supplier.

I_e : the interest earned per monetary unit per year.

Note : $I_c > I_e$

Q : the order quantity (a decision variable)

$\theta(t)$: deterioration with respect to time

$\theta(t) = \alpha\beta t^{\beta-1}$ where α denotes the scale parameter; $0 < \alpha < 1$.

β denotes the shape parameter; $\beta > 1$ i.e. deterioration increases with respect to time t.

$I(t)$: the inventory level at any instant of time t, $0 \leq t \leq T$.

T : the replenishment cycle time (a decision variable).

$K(T)$: the total cost per time unit of an inventory system.

The total cost of inventory system comprises of: (a) ordering cost; OC, (b) cost due to deterioration; DC, (c) inventory holding cost excluding interest charges; IHC, (d) interest charged on unsold item after the allowable trade credit when M < T; I_c, and minus (e) interest earned on revenue generated during the allowable permissible delay period; I_e.

Assumptions

1. The inventory system under consideration deals with the single item.
2. The planning horizon is infinite.
3. The demand of the product is decreasing function of the time.
4. Shortages are not allowed and lead-time is zero.
5. The units in inventory deteriorate with respect to time. The deteriorated units can neither be repaired nor replaced during the cycle time.
6. The retailer can deposit generated sales revenue in an interest bearing account during the allowable credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day-to-day expenses, and starts paying the interest charges on the unsold items in the inventory system.

Mathematical model

The inventory level; $I(t)$ depletes to meet the demand and deterioration of units. The rate of change of inventory level can be described by the following differential equation :

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -R(t) , \qquad 0 \leq t \leq T \qquad (1)$$

with the initial condition $I(0) = Q$ and the boundary condition $I(T) = 0$. Consequently, the solution of (1) is given by

$$I(t) = -\left[\int_0^t a(1-bt)e^{\alpha t^\beta} dt + X\right] e^{-\alpha t^\beta} \qquad 0 \leq t \leq T \qquad (2)$$

where X denotes constant of integration. The solution is obtained using series expansion of exponential and neglecting α^2 and its higher powers because $0 < \alpha < 1$. Using $I(0) = Q$, the order quantity is

$$Q = I(0) = \frac{a}{8(1+\beta)(2+\beta)(1+2\beta)} \begin{bmatrix} 16T + 72\alpha T^{1+\beta} + 32\alpha^2 T^{1+2\beta} - 28T^2\beta^3 - 28bT^2\beta - \\ 8b\alpha^2 T^{2\beta+2} - 8b\alpha T^{\beta+2} + 56T\beta + 56T\beta^2 + 16T\beta^3 - 8bT^2 \\ -10b\alpha^2 T^{2\beta+2} - 16b\alpha T^{\beta+2}\beta^2 - 24b\alpha T^{\beta+2}\beta \end{bmatrix} \qquad (3)$$

The total cost of inventory system per time unit consists of the following cost components:

a) Ordering cost; $OC = \dfrac{A}{T}$ (4)

b) Cost due to deterioration; DC per time unit; $DC = \frac{C}{T}\left[Q - \int_0^T R(t)dt\right]$

$$= \frac{C}{T}\left[\frac{a}{8(1+\beta)(2+\beta)(1+2\beta)}\begin{bmatrix}16T + 72\alpha T^{1+\beta} + 32\alpha^2 T^{1+2\beta} - 28T^2\beta^3 - 28bT^2\beta - \\ 8b\alpha^2 T^{2\beta+2} - 8b\alpha T^{\beta+2} + 56T\beta + 56T\beta^2 + 16T\beta^3 - 8bT^2 \\ -10b\alpha^2 T^{2\beta+2} - 16b\alpha T^{\beta+2}\beta^2 - 24b\alpha T^{\beta+2}\beta\end{bmatrix} + \frac{abT^2}{2} - aT\right]$$

(5)

c) Inventory holding cost; IHC per time unit;

$$IHC = \frac{h}{T}\int_0^T I(t)dt \qquad (6)$$

Regarding interest charges and earned, two cases may arise based on the lengths of the allowable credit period M and cycle time T.

Case1: M $\leq$ T.

Under the assumption mention in (6) (section 2.2) above, the retailer sells R(M)M units by the end of the permissible tread credit M and has to pay dues CR(M)M to the supplier. For the unsold items in the retailer's warehouse, the supplier charges an interest rate I_c during interval $[M, T]$. Hence, the interest charged, IC_1 per time unit is

$$IC_1 = \frac{CI_c}{T}\int_M^T I(t)dt \qquad (7)$$

During [0, M], the retailer sells the product and deposits the generated revenue into an interest earning account at the rate I_e per monetary unit per annum. Hence, the interest earned, IE_1 during [0, M] per time unit is

$$IE_1 = \frac{PI_e}{T}\int_0^M R(t)tdt = \frac{PI_e}{T}\left[\frac{1}{2}aM^2 - \frac{1}{3}bM^3\right] \qquad (8)$$

Hence, the total cost; $K_1(T)$ of an inventory system per time unit is

$$K_1(T) = OC + DC + IHC + IC_1 - IE_1 \qquad (9)$$

Case2: T $\leq$ M

In this scenario, the retailer sells R(T)T- units in all by the end of the cycle time and has CR(T)T amount in his account to pay the supplier in full by the end of the credit Period M. Hence the interest charges $IC_2 = 0$ (10)

and the interest earned per time unit is

$$IE_2 = \frac{PI_e}{T}\left[\int_0^T R(t)tdt + R(T)T(M-T)\right]$$

$$= \frac{PI_e}{T}\left[\frac{-abT^3}{3} + \frac{aT^2}{2} + aTM - aT^2 - abT^2M + abT^3\right] \qquad (11)$$

Hence, the total cost; $K_2(T)$ of an inventory system per time unit is

$$K_2(T) = OC + DC + IHC + IC_2 - IE_2 \qquad (12)$$

As a result, the total cost; $K(T)$ of an inventory system per time unit is

$$K(T)=\begin{cases}K_1(T), M\le T\\ K_2(T), M\ge T\end{cases} \tag{13}$$

For T = M, we have $K_1(M)=K_2(M)$. (14)

Analytical Results

For M $\le$ T, the value of T can be obtained by solving

$$\frac{\partial K_1(T)}{\partial T}= 0 \tag{15}$$

The obtained $T=T_1$ (say) minimizes the total cost provided

$$\frac{\partial^2 K_1(T)}{\partial T^2}>0 \qquad \text{for } T=T_1 \tag{16}$$

For M > T, the value of $T=T_2$ (say) can be obtained by solving

$$\frac{\partial K_2(T)}{\partial T}=0 \tag{17}$$

The obtained $T=T_2$ minimizes the total cost per time unit of an inventory system, provided

$$\frac{\partial^2 K_2(T)}{\partial T^2}>0 \qquad \text{for } T=T_2 \tag{18}$$

Computational algorithm

The outline to obtain optimal solution for retailer is as follows:

Step1: Initialize all the parametric values.

Step2: Compute T_1 from eq.(15). If $M<T_1$ then $K_1(T_1)$ (eq. 9) gives the minimum cost else go to step3.

Step3: Compute T_2 from eq. (17). If $M>T_2$ then $K_2(T_2)$ (eq.12) gives minimum cost for retailer; else go to step 4.

Step4: $K_1(M)=K_2(M)$ (eq. 14) is the minimum cost.

Step5: stop.

Numerical examples

Example1: Consider the following parametric values in proper units:

$$[a,b,C,P,h,A,I_c,I_e,M,\alpha,\beta]=[300,0.2,20,40,1,200,12\%,9\%,30/365,0.3,3.5]$$

Then $T_1=0.5270$ years which is less than M. So $K_1(T_1)=\$612.86$ is minimum cost (see Fig.1) procuring 150.79 units. Also $\frac{\partial^2 K_1(T)}{\partial T^2}=3878.12>0$ proves convexity of the total cost.

Example2: for parametric values :

$$[a,b,C,P,h,A,I_e,M,\alpha,\beta]=[2500,0.1,28,40,1,250,12\%,60/365,0.3,1.8]$$

Then $T_2 = 0.1591$ years which is greater than M. Using algorithm $K_2(T_2) = \$974.61$ is the minimum cost (see Fig.2) for purchase of 396.17 units. Also $\frac{\partial^2 K_2(T_2)}{\partial T_2^{\ 2}} = 137787 > 0$ shows that the total cost per time unit of an inventory system is minimum.

Using parametric values defined in example 1, sensitivity analysis is performed by changing values as -40%, -20%, 20%, 40% on decision variable and the objective function.

Table1: ***sensitivity analysis***

Parameter	% changes			
		T	Q	K_1
b	-40	-1.85	1.24	0.36
	-20	-0.94	0.63	0.18
	20	1.01	-0.65	-0.17
	40	2.06	-1/32	-0.32
M	-40	0.19	3.83	-0.46
	-20	0.09	1.93	0.10
	20	-0.13	-1.94	-0.12
	40	-0.26	-3.91	-0.25
α	-40	6.64	-3.00	6.07
	-20	2.97	-1.42	2.72
	20	-2.50	1.29	-2.29
	40	-4.74	2.49	-4.26
β	-40	-13.43	15.46	-11.77
	-20	-6.03	5.41	-5.33
	20	4.93	-3.20	4.41
	40	9.01	-5.21	8.07

It is observed that increase in deterioration rate decreases cycle time because retailer will have to put order frequently. The more deterioration rate of units increases total cost of an inventory system. To reduce deterioration rate of units he replenish order of smaller size. Increase in shape parameter of deterioration with respect to time increases cycle time and decreases procurement quantity significantly. The total cost is of an inventory system is very sensitive to shape parameter. The permissible delay period decrease cycle time and total cost. Decrease in total cost is due to the fact that the retailer can earn interest on generated sales revenue for a longer period. Increase in the declining demand rate directs retailer to put order of smaller size after a long time and decrease in total cost.

Conclusions

The effect of time dependent deterioration of units in inventory is studied when the demand for the product is decreasing and the supplier offers the retailer a credit period to settle the account. It is established that the retailer should replenish smaller order more frequently to avail of sales promotional tool as trade credit. The study is interesting by allowing partial backlogging, inflation, stochastic demand etc.

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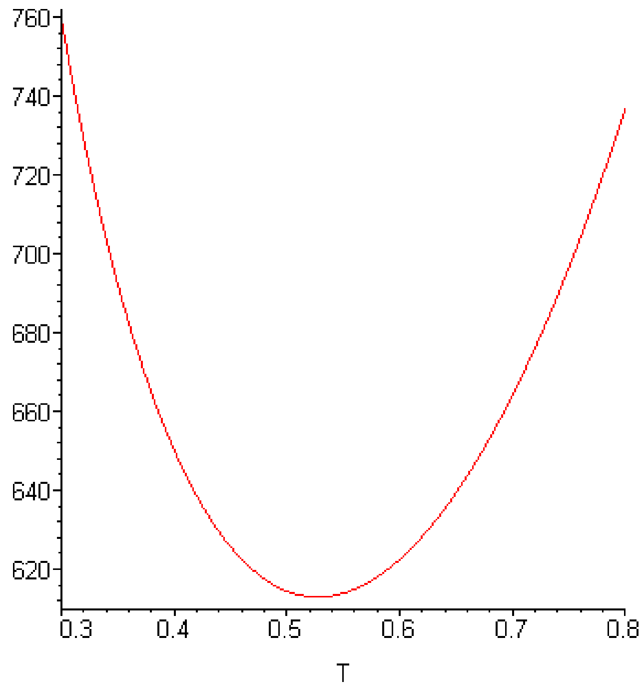

Fig 1: *Convexity of total cost when* $M \leq T$

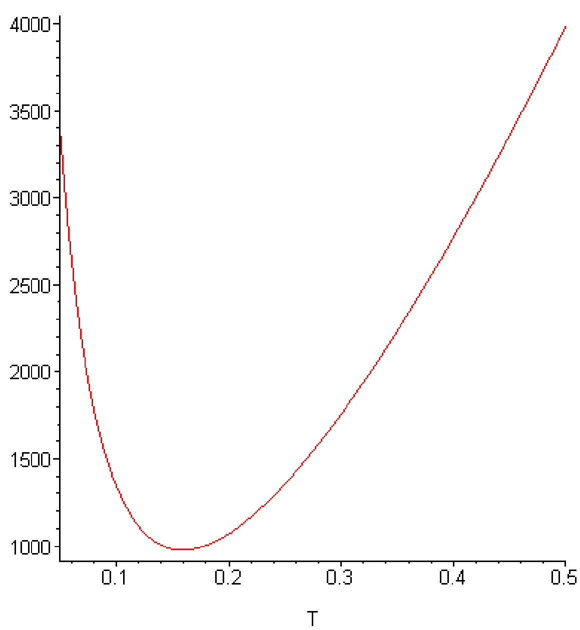

Fig 2: *Convexity of total cost when* $M > T$